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FREE VIBRATIONS OF PIEZOELECTRIC CYLINDRICAL SHELLS FILLED WITH COMPRESSIBLE FLUID

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Abstract—Three displacement functions are introduced to represent each mechanical displacement according to the three-dimensional theory. After expanding these functions and the electric potential with orthogonal series, the free vibration equation of piezoelectric cylindrical shells satisfying SS3 edge conditions can be obtained. The equation was solved by utilizing Bessel functions with complex arguments. The effects of compressible fluid on shells are considered and some new phenomena which are exclusive for piezoelectric cylindrical shells are reported. Results of empty infinite piezoelectric cylindrical shells are compared to those presented in relative references. Some lowest frequencies that were missed by Paul and Venkatesan [Paul, H. S. and Venkatesan, M. (1987). Vibrations of hollow circular cylinder of piezoelectric ceramics. *Journal of the Acoustic Society of America* **82**, 852–856] were discovered. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

Because of the particular mechanical–electrical coupling effect, piezoelectric phenomenon has always attracted many attentions in both theoretical and engineering science since the Curie brothers' studies initiated in 1880. However, a great number of problems of piezoelectric materials that had been solved up to the 1950s belongs to static problems because of its complex coupling effect between mechanics and electricity. Dokmeci (1980) in his review article, stressed the importance on wave and vibration in piezoelectric crystal bars, rings, disks, laminae and in particular, plates and shells. Many investigations on dynamic responses of various piezoelectric structures have been done in these areas by using various shell and plate theories.

Vibrations of piezoelectric materials have attracted many attentions since the fifties. Haskins and Walsh (1956) first studied the axisymmetric free vibrations of radially polarized piezoelectric cylindrical shells with transverse isotropy by adopting a classical shell theory, though some mechanical quantities could not be satisfied exactly during the separation of basic equations; in the case of very small thickness of the shell, their results were coincident with the experiment well. The plate and shell theories suggested by Drumheller and Kalnins (1969) did satisfy exactly the basic equations both of mechanics and electricity; however, iterative technology should be used to solve relative equations because they were still coupled. Tzou and Zhong (1994) recently developed a linear shell theory, which can be used to derive the approximate controlling equations of vibrations of thin or moderately thick piezoelectric shells, but numerical example was not given. Babaev and Savin (1988), based on Kirchhoff-Love hypotheses, solved the nonsteady hydroelasticity of coaxial piezoceramic cylindrical shells during electrical excitation. Babaev *et al.* (1990) studied the transient vibrations of thin-walled, radially polarized, piezoelectric cylindrical shells coupled with fluid.

Earlier investigations by the methods of three dimensional theory concentrated on the axisymmetric and radial vibrations of cylinders and thin circular rings, such as Stephenson

(1956a, 1956b) and Adelman *et al.* (1974, 1975). Paul (1966) first derived the frequency equation of piezoelectric cylindrical shell without giving numerical results. Paul and Venkatesan (1987) employed Paul's method [Paul (1966)] to obtain the natural frequencies of infinite piezoelectric cylindrical shells with either shorted electroded or free lateral surfaces; however, some frequencies were missed in their calculation. Studies on finite cylinders or cylindrical shells are much less. Paul and Natarajan (1994a, 1994b) investigated the flexural and axisymmetric free vibrations of finite free piezoelectric cylindrical shells with free edges ($\sigma_r = \tau_{rz} = \tau_{\theta z} = 0$ at both edges, here σ_r, τ_{rz} and $\tau_{r\theta}$ are stress components in circular coordinates), but their displacement functions could not satisfy the edge conditions completely and therefore approximation was inevitably introduced. Free vibration of finite fluid-filled piezoelectric cylindrical shells has not been found by authors, though similar research on transversely isotropic cylindrical shell has been made by Jain (1974), where shell theory was employed.

In this paper, three displacement functions are first introduced to represent the components of displacement [Ding Haojiang *et al.* (1996)]. Then the displacement functions and the electric potential are expanded in terms of orthogonal series both in θ and z directions and the SS3 edge conditions are automatically satisfied ($U_z = \tau_{rz} = \tau_{\theta z} = \varphi = 0$, here U_z is the displacement component in z direction and φ is the electric potential). Substituting these expressions into the basic equations of piezoelectricity, the controlled equations of free vibration problems are obtained in terms of displacement functions and electric potential. One is a Bessel equation only of displacement function ψ, the other is a coupled second-order ordinary differential equation set of displacement functions G and W and electric potential φ. Although the Bessel function with complex argument may be accounted in the solving procedure, our investigation reveals that real frequency still exists. Considering the effect of filled compressible fluid, the variations of the smallest frequency versus wave number and thickness are studied.

2. SIMPLIFICATION OF BASIC EQUATIONS

In circular coordinates (r, θ, z), the mechanical displacements of transversely isotropic piezoelectric materials can be expressed by three displacement functions as follows [Ding Haojiang *et al.* (1996)]

$$\begin{cases} U_r = \dfrac{1}{r}\dfrac{\partial \psi}{\partial \theta} - \dfrac{\partial G}{\partial r} \\ U_\theta = -\dfrac{\partial \psi}{\partial r} - \dfrac{1}{r}\dfrac{\partial G}{\partial \theta} \\ U_z = W \end{cases} \tag{1}$$

where U_r, U_θ and U_z are components of displacement in r, θ and z directions, respectively, ψ, G and W are displacement functions that satisfy the following equations, respectively:

$$\begin{cases} \left(c_{66}\nabla_1^2 + c_{44}\dfrac{\partial^2}{\partial z^2} - \rho\dfrac{\partial^2}{\partial t^2}\right)\psi = 0 \\ \left(c_{11}\nabla_1^2 + c_{44}\dfrac{\partial^2}{\partial z^2} - \rho\dfrac{\partial^2}{\partial t^2}\right)G - (c_{13}+c_{44})\dfrac{\partial W}{\partial z} - (e_{15}+e_{31})\dfrac{\partial \varphi}{\partial z} = 0 \\ -(c_{13}+c_{44})\nabla_1^2\dfrac{\partial G}{\partial z} + \left(c_{44}\nabla_1^2 + c_{33}\dfrac{\partial^2}{\partial z^2} - \rho\dfrac{\partial^2}{\partial t^2}\right)W + \left(e_{15}\nabla_1^2 + e_{33}\dfrac{\partial^2}{\partial z^2}\right)\varphi = 0 \\ (e_{15}+e_{31})\nabla_1^2\dfrac{\partial G}{\partial z} - \left(e_{15}\nabla_1^2 + e_{33}\dfrac{\partial^2}{\partial z^2}\right)W + \left(\varepsilon_{11}\nabla_1^2 + \varepsilon_{33}\dfrac{\partial^2}{\partial z^2}\right)\varphi = 0. \end{cases} \tag{2}$$

In the above equations, c_{ij} are elastic constants of which only five are independent, i.e.

$c_{66} = (c_{11} - c_{12})/2$; e_{ij}, ε_{ij}, ρ and φ are piezoelectric constants, dielectric constants, density and electrical potential, respectively, and

$$\nabla_1^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}. \tag{3}$$

The SS3 edge conditions [Hoff and Soong (1965)] are

$$U_z = \tau_{rz} = \tau_{\theta z} = 0 = \varphi \quad (z = 0, L) \tag{4}$$

where L is the length of piezoelectric cylindrical shells. The stress components and electrical displacements expressed by mechanical displacements and electrical potential can be found in Paul (1966).

Displacement functions can be taken in the following forms:

$$\begin{cases} \psi(r,\theta,z,t) = \psi(r)\cos(m\pi\zeta)\sin(n\theta)\cdot e^{i\omega t} \\ G(r,\theta,z,t) = G(r)\cos(m\pi\zeta)\cos(n\theta)\cdot e^{i\omega t} \\ W(r,\theta,z,t) = [W(r)/R]\sin(m\pi\zeta)\cos(n\theta)\cdot e^{i\omega t} \\ \varphi(r,\theta,z,t) = [c_{44}\varphi(r)/(R\cdot e_{33})]\sin(m\pi\zeta)\cos(n\theta)\cdot e^{i\omega t} \end{cases} \tag{5}$$

where $\zeta = z/L$. Substituting eqn (5) into eqn (2) and adopting the non-dimensional procedure, one obtains

$$(\bar{c}_{66}\nabla_2^2 + \Omega^2 - r_m^2)\psi = 0 \tag{6}$$

$$\begin{bmatrix} \bar{c}_{11}\nabla_2^2 + \Omega^2 - r_m^2 & -(\bar{c}_{13}+1)r_m & -(\bar{e}_{15}+\bar{e}_{31})r_m \\ (\bar{c}_{13}+1)r_m\nabla_2^2 & \nabla_2^2 + \Omega^2 - \bar{c}_{33}r_m^2 & \bar{e}_{15}\nabla_2^2 - r_m \\ -(\bar{e}_{15}+\bar{e}_{31})r_m\nabla_2^2 & -(\bar{e}_{15}\nabla_2^2 - r_m^2) & k_{13}^{-2}\nabla_2^2 - k_{33}^{-2}r_m^2 \end{bmatrix} \begin{Bmatrix} G \\ W \\ \varphi \end{Bmatrix} = 0 \tag{7}$$

where $\nabla_2^2 = (1/\xi)(d/d\xi)(\xi d/d\xi) - n^2/\xi^2$ and the non-dimensional parameters are introduced as

$$\xi = r/R;\ r_m = m\pi R/L;\ \Omega = \omega/\omega_s;\ \omega_s = v_2/R;\ R = (a+b)/2$$
$$v_2 = \sqrt{c_{44}/\rho};\ \bar{c}_{ij} = c_{ij}/c_{44};\ \bar{e}_{ij} = e_{ij}/e_{33};\ k_{i3}^2 = e_{33}^2/(\varepsilon_{ii}\cdot c_{44}) \tag{8}$$

where a and b are inner and outer radii of the cylindrical shell, respectively.

Equation (6) is a Bessel equation, one can write out its solution directly

$$\psi(\xi) = A_4 J_n(k_4\xi) + D_4 Y_n(k_4\xi) \tag{9}$$

where $k_4^2 = (\Omega^2 - r_m^2)/\bar{c}_{66}$; $J_n(\cdot)$ and $Y_n(\cdot)$ are the nth order Bessel functions of the first and second kind, respectively, A_4, D_4 are arbitrary constants.

Suppose that one set of solution to eqn (7) be

$$\begin{Bmatrix} G_1 \\ W_1 \\ \varphi_1 \end{Bmatrix} = J_n(k\xi)\begin{Bmatrix} A \\ B \\ C \end{Bmatrix} \tag{10}$$

where A, B and C are arbitrary constants. The determinant equation can be obtained by substituting the above equation into eqn (7)

$$\begin{vmatrix} \bar{c}_{11}k^2-\Omega^2+r_m^2 & (\bar{c}_{13}+1)r_m & (\bar{e}_{15}+\bar{e}_{31})r_m \\ (\bar{c}_{13}+1)r_mk^2 & k^2-\Omega^2+\bar{c}_{33}r_m^2 & \bar{e}_{15}k^2+r_m^2 \\ (\bar{e}_{15}+\bar{e}_{31})r_mk^2 & \bar{e}_{15}k^2+r_m & -k_{13}^{-2}k^2-k_{33}^{-2}r_m^2 \end{vmatrix} = 0. \tag{11}$$

Three roots of $k^2(k_1^2, k_2^2$ and $k_3^2)$ can be obtained from eqn (11), among which at least one must be real, denoting it as k_3^2. In general, one appreciates three distinct roots. In addition, zero roots correspond to spurious frequencies that should be neglected here and other methods should be employed when equivalent roots emerge in eqn (11).

Let $\mathrm{Re}[k_i] \geqslant 0$, one can obtain a set of solution to eqn (7)

$$\begin{Bmatrix} G_1 \\ W_1 \\ \varphi_1 \end{Bmatrix} = \sum_{i=1}^{3} J_n(k_i\xi)\begin{Bmatrix} A_i \\ B_i \\ C_i \end{Bmatrix} \tag{12}$$

where A_i, B_i and C_i satisfy

$$\begin{cases} B_i = d_iA_i \\ C_i = f_iA_i \end{cases} \tag{13}$$

and d_i and f_i are determined by

$$\begin{bmatrix} (\bar{c}_{13}+1)r_m & (\bar{e}_{15}+\bar{e}_{31})r_m \\ k_i^2-\Omega^2+\bar{c}_{33}r_m^2 & \bar{e}_{15}k_i^2+r_m^2 \end{bmatrix}\begin{Bmatrix} d_i \\ f_i \end{Bmatrix} = \begin{Bmatrix} -(\bar{c}_{11}k_i^2-\Omega^2+r_m^2) \\ -(\bar{c}_{13}+1)r_mk_i^2 \end{Bmatrix}. \tag{14}$$

Obviously, the other independent set of solution to eqn (7) is

$$\begin{Bmatrix} G_2 \\ W_2 \\ \varphi_2 \end{Bmatrix} = \sum_{i=1}^{3} Y_n(k_i\xi)\begin{Bmatrix} D_i \\ E_i \\ F_i \end{Bmatrix} \tag{15}$$

and

$$\begin{cases} E_i = d_iD_i \\ F_i = f_iD_i \end{cases}. \tag{16}$$

The complete solution to be eqn (7) can be obtained by combining expressions (12) and (15)

$$\begin{Bmatrix} G \\ W \\ \varphi \end{Bmatrix} = \begin{Bmatrix} G_1 \\ W_1 \\ \varphi_1 \end{Bmatrix} + \begin{Bmatrix} G_2 \\ W_2 \\ \varphi_2 \end{Bmatrix} = \sum_{i=1}^{3} \begin{bmatrix} J_n(k_i\xi) & Y_n(k_i\xi) \\ d_iJ_n(k_i\xi) & d_iY_n(k_i\xi) \\ f_iJ_n(k_i\xi) & f_iY_n(k_i\xi) \end{bmatrix}\begin{Bmatrix} A_i \\ D_i \end{Bmatrix}. \tag{17}$$

Thus, the exact solution to displacement functions and electric potential is obtained. The frequency equation can be derived by substituting expressions (9) and (17) into constitutive equations and considering the boundary conditions. The non-dimensional expressions of stress components are

$$\begin{cases} \bar{\sigma}_r = (1/R^2)\left\{-\sum_{i=1}^{3}[\bar{c}_{11}k_i^2P_i''(k_i\xi)+\bar{c}_{12}k_iP_i'(k_i\xi)/\xi-(\bar{c}_{12}n^2/\xi^2+\bar{c}_{13}r_md_i+\bar{e}_{31}r_mf_i)P_i(k_i\xi)]\right. \\ \left. \quad +n(\bar{c}_{11}-\bar{c}_{12})[k_4P_4'(k_4\xi)/\xi-P_4(k_4\xi)/\xi^2]\right\}\cos(m\pi\zeta)\cos(n\theta)e^{i\omega t} \\ \bar{\tau}_{rz} = (1/R^2)\sum_{i=1}^{3}[k_i(r_m+d_i+\bar{e}_{15}f_i)P_i'(k_i\xi)]-nr_mP_4(k_4\xi)/\xi]\sin(m\pi\zeta)\cos(n\theta)e^{i\omega t} \\ \bar{\tau}_{r\theta} = (\bar{c}_{66}/R^2)\left\{\sum_{i=1}^{3}2n[k_iP_i'(k_i\xi)/\xi-P_i(k_i\xi)/\xi^2]-k_4^2P_4''(k_4\xi)+k_4P_4'(k_4\xi)/\xi\right. \\ \left. \quad -n^2P_4(k_4\xi)/\xi^2\right\}\cos(m\pi\zeta)\sin(n\theta)e^{i\omega t} \end{cases} \tag{18}$$

where the prime denotes differentiation with respect to $x(x = k_i\xi)$ and

$$\begin{cases} \bar{\sigma}_r = \sigma_r/c_{44}, \quad \bar{\tau}_{rz} = \tau_{rz}/c_{44}, \quad \bar{\tau}_{r\theta} = \tau_{r\theta}/c_{44} \\ P_i(x) = A_iJ_n(x)+D_iY_n(x), \quad (i = 1,2,3,4). \end{cases} \tag{19}$$

3. COUPLING EFFECT OF FLUID AND SHELL

The basic equation of compressible fluid in cylindrical coordinates is known as

$$\left(\frac{1}{c_f^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)\Phi = 0 \tag{20}$$

where,

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z^2};$$

c_f is the sound velocity in fluid. $\Phi(r,\theta,z,t)$ is the velocity potential which can be expanded in the following terms:

$$\Phi(r,\theta,z,t) = \Phi(r)\cos(m\pi\zeta)\cos(n\theta)e^{i\omega t} \tag{21}$$

Substituting (21) into (20) gives

$$\frac{d^2\Phi}{d\xi^2}+\frac{1}{\xi}\frac{d\Phi}{d\xi}+\left[\left(\frac{\Omega^2}{c_0^2}-r_m^2\right)-\frac{n^2}{\xi^2}\right]\Phi = 0. \tag{22}$$

Take account of the finite value condition of Φ at $r = 0$, the solution of eqn (22) is taken in the following form:

$$\Phi(\xi) = DJ_n(\gamma\xi) \tag{23}$$

for $\gamma^2 > 0$ where $\gamma^2 = \Omega^2/c_0^2-r_m^2$, while for $\gamma^2 < 0$ $J_n(\cdot)$ should be replaced with $I_n(\cdot)$, the modified Bessel function of the first kind, in eqn (23) and hereafter, $c_0 = c_f/v_2$, D is an arbitrary constant.

From the Bernoulli equation, the dynamic pressure and the velocity of fluid can be obtained as follows:

$$p_f = iD\frac{\Omega\rho_0}{v_2 R}J_n(\gamma\xi)\cos(m\pi\zeta)\cos(n\theta)e^{i\omega t} \tag{24}$$

$$v_f^* = \frac{D}{R}\cdot\frac{\mathrm{d}}{\mathrm{d}\xi}[J_n(\gamma\xi)]\cos(m\pi\zeta)\cos(n\theta)e^{i\omega t} \tag{25}$$

where ρ_f is the fluid density and $\rho_0 = \rho_f/\rho$. Neglect the effects of viscous and static pressure of fluid, the boundary conditions at the interaction surface are

$$\bar{\sigma}_r = -p_f;\ \bar{\tau}_{r\theta} = \bar{\tau}_{rz} = 0;\ v_r = v_r^*. \tag{26}$$

It is known that $v_r = \partial U_r/\partial t$, thus from eqn (23)–(25), we can obtain

$$\bar{\sigma}_r = -\Omega^2\rho_0 H(\gamma\xi)\bar{U}_r/\gamma \quad (\xi = \xi_a = 1 - t_1/2) \tag{27}$$

where

$$\begin{cases} H(x) = J_n(x)/J_n'(x) \\ t_1 = (b-a)/R;\ \bar{U}_r = U_r/R \end{cases} \tag{28}$$

4. FREQUENCY EQUATION

Suppose that the electric potential be zero at the inner and outer surfaces. Considering the boundary condition expressions derived in the previous section

$$\begin{cases} \bar{\sigma}_r = -\Omega^2\rho_0 H(\gamma\xi)\bar{U}_r;\ \bar{\tau}_{r\theta} = \bar{\tau}_{rz} = 0 = \varphi & (\xi = \xi_a = 1 - t_1/2) \\ \bar{\sigma}_r = \bar{\tau}_{r\theta} = \bar{\tau}_{rz} = 0 = \varphi & (\xi = \xi_b = 1 + t_1/2). \end{cases} \tag{29}$$

It is noticed that boundary conditions of empty shells can be derived by setting $\rho_0 = 0$ in eqn (29).

The frequency equation can be derived by substituting eqn (18) and the expression of φ in eqn (17) into (29) as follows

$$|E_{\alpha\beta}| = 0 \quad (\alpha, \beta = 1, 2, \ldots 8) \tag{30}$$

where

$$\begin{cases} E_{1(2j-1)} = -\bar{c}_{11}k_j^2 J_n''(k_j\xi_a) - \bar{c}_{12}k_j J_n'(k_j\xi_a)/\xi_a \\ \quad + [\bar{c}_{12}n^2/\xi_a^2 + r_m(\bar{c}_{13}d_j + \bar{e}_{31}f_j)]J_n(k_j\xi_a) - Q(\xi_a)k_j J_n'(k_j\xi_a) \\ E_{17} = n(c_{11} - c_{12})[k_4 J_n'(k_4\xi_a)/\xi_a - J_n(k_4\xi_a)/\xi_a^2] + Q(\xi_a)nJ_n(k_4\xi_a)/\xi_a \\ E_{2(2j-1)} = k_j(r_m + d_j + \bar{e}_{15}f_j)J_n'(k_j\xi_a) \\ E_{27} = -nr_m J_n(k_4\xi_a) \quad (j = 1, 2, 3) \\ E_{3(2j-1)} = 2n[k_j J_n'(k_j\xi_a)/\xi_a - J_n(k_j\xi_a)/\xi_a^2] \\ E_{37} = -k_4^2 J_n''(k_4\xi_a) + k_4 J_n'(k_4\xi_a)/\xi_a - n^2 J_n(k_4\xi_a)/\xi_a^2 \\ E_{4(2j-1)} = f_j J_n(k_j\xi_a) \quad E_{47} = 0 \end{cases} \tag{31}$$

where, only the $(2j-1)$th column $(j = 1, 2, 3, 4)$ of the first four rows of matrix $[E_{\alpha\beta}]$ are presented. The elements of $2j$th column $(j = 1, 2, 3, 4)$ can be obtained by changing the Bessel functions of the first kind in the $(2j-1)$th column into the Bessel functions of the second kind, and the elements of $(i+4)$th row $(i = 1, 2, 3, 4)$ can be obtained by replacing ξ_a in the ith row with ξ_b. And

$$Q(\xi) = \begin{cases} \rho_0 \Omega^2 H(\gamma\xi)/\gamma & (\xi = \xi_a) \\ 0 & (\xi = \xi_b) \end{cases} \tag{32}$$

5. NUMERICAL RESULTS AND DISCUSSIONS

The piezoelectric material PZT4 is to be considered in the following calculations, and the fluid considered here is water. The relative non-dimensional parameters are

$$\bar{c}_{11} = 5.4297 \quad \bar{c}_{12} = 3.0391 \quad \bar{c}_{13} = 2.9023 \quad \bar{c}_{33} = 4.4922$$

$$\bar{e}_{31} = -0.3444 \quad \bar{e}_{15} = 0.8411 \quad k_{13}^2 = 1.3787 \quad k_{33}^2 = 1.5848$$

$$\rho_0 = \begin{cases} 0 & \text{for empty shells} \\ 0.1333 & \text{for fluid-filled shells} \end{cases}$$

$$c_0 = 0.9874$$

Example 1 : *free vibrations of piezoelectric cylindrical shells with SS3 edge conditions*

Figure 1 displays curves of the lowest non-dimensional natural frequency Ω vs the thickness-to-mean radius ratio t_1 for two values of non-dimensional wave number $r_m = 1.0$ and $r_m = 2.0$ in both cases of empty and fluid-filled piezoelectric cylindrical shell, where solid lines corresponding to the empty case while dotted lines corresponding to the fluid-filled case. It is seen that the smaller t_1 is, the stronger the effect of compressible fluid on Ω is and it decreases with the increase of t_1. For flexural vibrations ($n = 1, 2$), the effect of compressible fluid on Ω is certainly small when t_1 is very big. This phenomenon is similar to that of transversely isotropic cylindrical shells [Jain (1974)]. However, great difference exists between them, i.e. the effect of compressible fluid on the axisymmetric vibration ($n = 0$, it is also known as the breathing mode) is significant and does not decrease greatly with the increase of t_1. This is the special phenomenon occupied by the piezoelectric cylindrical shells and has not been reported before. It is therefore suggested that the effect of fluid on the breathing mode, due to the particular mechanical–electrical coupling effect of the piezoelectric media, should be the most significant.

Figure 2 displays curves of non-dimensional frequency Ω vs non-dimensional wave number r_m for two values of thickness-to-mean radius ratio $t_1 = 0.5$ and $t_1 = 1.0$. It is shown that the frequency Ω increases with the increase of r_m. For flexural vibrations ($n = 1, 2$), the effect of compressible fluid is always small. It is somewhat different for the axisymmetric case ($n = 0$), i.e. there exists a critical wave number ($r_m^* \approx 1.20$), when $r_m < r_m^*$, the effect increases with the increase of Ω while it decreases when $r_m > r_m^*$. This is another special phenomenon occupied by the piezoelectric cylindrical shells. It is also due to the particular mechanical–electrical coupling effect of the piezoelectric media.

Example 2 : *free vibrations of empty infinite piezoelectric cylindrical shells*

Replacing the terms $\sin(m\pi\zeta)e^{i\omega t}$ and $\cos(m\pi\zeta)e^{i\omega t}$ in eqn (5) with $e^{i(\lambda z - \omega t)}$ and processing the same derivation as in Section 2, one can obtain the same determinant equation as eqn (11), but it should be noticed that r_m is defined as $r_m = \lambda R$ by now. Considering the same boundary conditions as expression (29), the frequency equation of infinite empty piezoelectric cylindrical shells with traction-free surfaces can be obtained. Here the detail derivations are not presented for brevity.

After substituting material constants listed above into eqn (11), it can be seen that $\Omega/r_m = 0.85116$ is a branch point of eqn (11), i.e. k_1^2 and k_2^2 are conjugate complexes in the solving zone $\Omega/r_m < 0.85116$, while they are real numbers in $\Omega/r_m \geqslant 0.85116$. In addition, $\Omega = 1.0r_m$ and $2.4662r_m$ are spurious frequencies which should be neglected in the calculations because zero roots exist at these points. The present results and some results published by Paul and Venkatesan (1987) are listed in Table 1.

In Table 1, data in parentheses are cited from Paul and Venkatesan (1987) and data prefixed with * are frequencies obtained by the present method but were not reported by

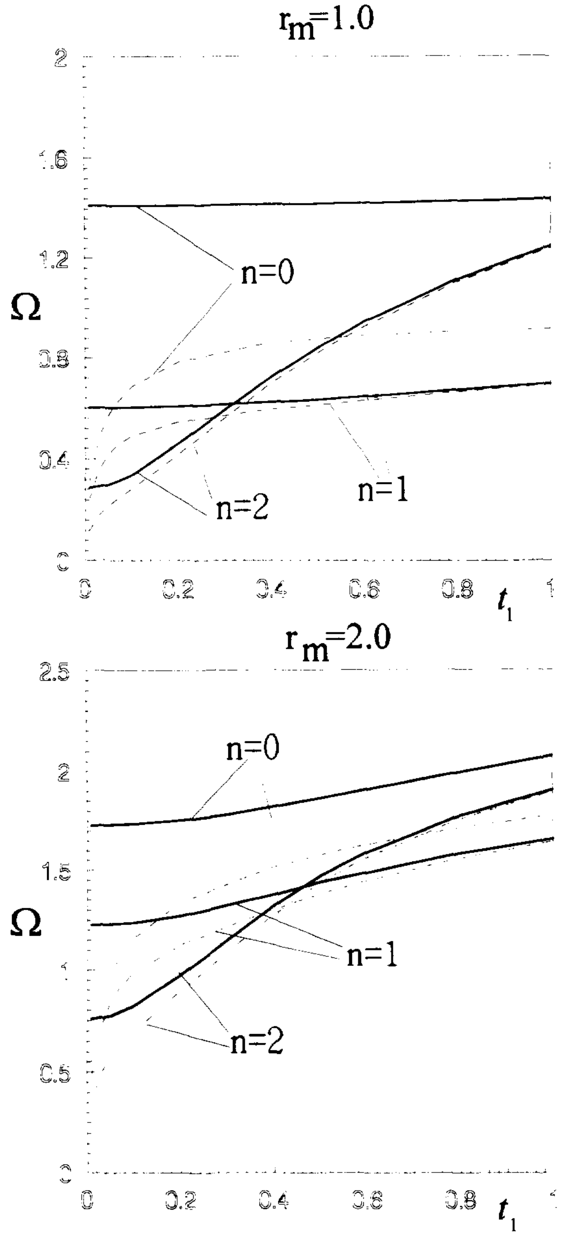

Fig. 1. Curves of non-dimensional frequency vs thickness-to-mean radius ratio.

Paul and Venkatesan (1987). "(· · ·)" denotes where they missed the lowest frequencies. It is noticed that all the missed frequencies belong to the solving zone $\Omega/r_m < 0.85116$, that is to say, the lowest frequencies in case of k_i^2 $(i = 1, 2)$ being conjugate complexes were missed by Paul and Venkatesan (1987). In fact, only two cases, $k_i^2 > 0$ and $k_i^2 < 0$, were computed for frequencies in their paper.

It is seen from Table 1 that present results agree with the results presented in Paul and Venkatesan (1987) quite well except for the missed frequencies. In addition, when $n = 2, r_m = 2.0$, we do not find a lowest frequency close to the value 1.74713 which was presented by Paul and Venkatesan (1987) during our calculations. By repeating the calculations in accordance with their formulae, we can only obtain the value 1.90402, which is also the result of present method.

6. CONCLUSIONS

(1) In the present paper, the frequency equation of piezoelectric cylindrical shells with SS3 edge conditions is derived based on the three-dimensional theory, including the effect

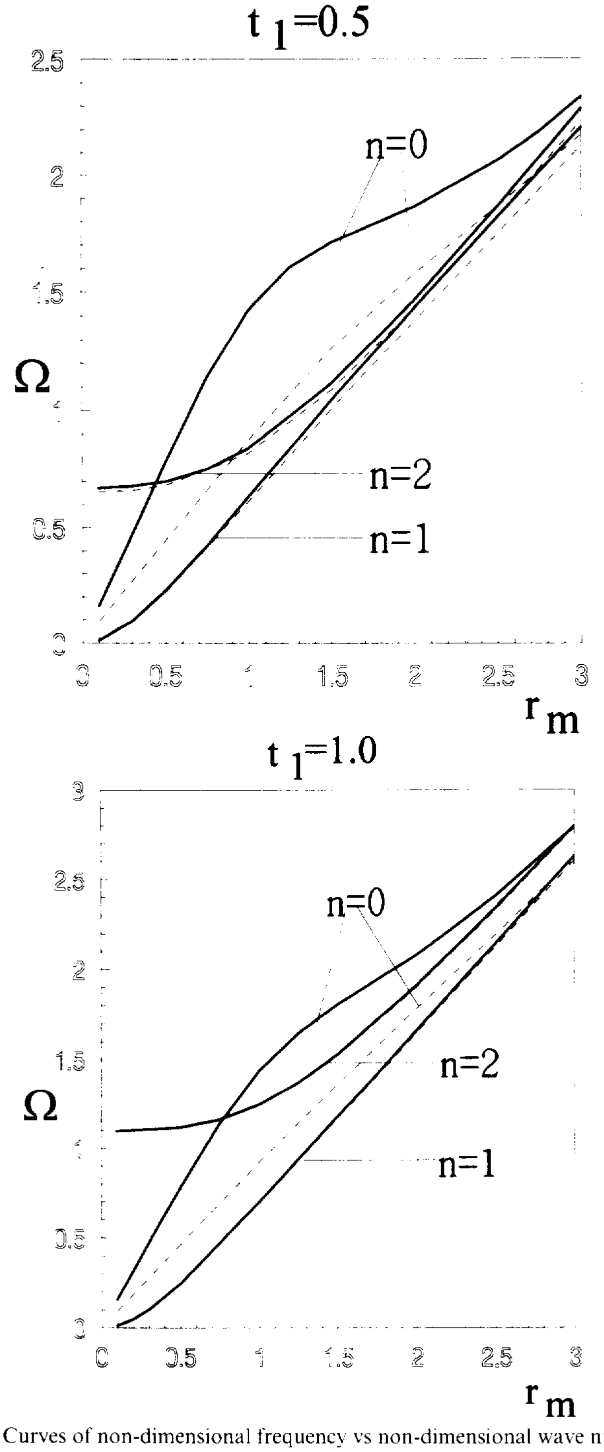

Fig. 2. Curves of non-dimensional frequency vs non-dimensional wave number.

of the filled compressible fluid. Results are presented to investigate the effects of wave number and thickness of cylindrical shell on the smallest frequency.

(2) Similar phenomenon exists for the effect of inner compressible fluid on flexural vibrations between piezoelectric and transversely isotropic cylindrical shell, but there is a great difference between them for the axisymmetric free vibrations. The compressible fluid has a significant effect on the smallest frequency of piezoelectric cylindrical shells for axisymmetric vibrations and it does not decrease greatly with t_1 increasing.

(3) The effect of compressible fluid on the axisymmetric free vibrations of piezoelectric cylindrical shell does not increase simply with the increase of the non-dimensional wave

Table 1. Free vibration frequencies Ω of infinite piezoelectric cylindrical shells ($t_1 = 1.0$)

r_m	$n = 0$	$n = 1$	$n = 2$	$n = 3$
0.01	*0.01582	*0.00013		
	2.21764	1.00478	1.09611	2.29474
	(···)	(···)	(1.09614)	(2.29463)
	(2.21722)	(1.00262)		
1.00		*0.69679		
	1.43728	1.74936	1.29820	2.32647
	(1.42886)	(···)	(1.30666)	(2.29478)
		(1.74682)		
2.00		*1.65614		
	2.07361	2.35572	1.90402	2.84573
	(2.05896)	(···)	(1.74713)?	(2.89942)
		(2.32415)		
3.00	2.79204	2.62435	2.78390	2.67739
	(2.77349)	(2.67795)	(2.75902)	(2.69625)

number. There actually exists a critical wave number $r_m^*(\approx 1.20)$, i.e. when $r_m < r_m^*$, the effect increases with the increase of Ω while decreases when $r_m > r_m^*$.

(4) The method proposed here obviously prevails in dealing with the free vibration problem of piezoelectric cylindrical shells by using the Bessel functions of complex argument directly and results show that it is more rational. The method can be generalized and applied to the analysis of free vibration of other cases like orthotropic cylinders or cylindrical shells and can be used to check the accuracy of various shell theories.

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